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OCEANOLOGY

Direct and Inverse Energy Cascades in the Ocean during Vortex Elongation

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Abstract—When mesoscale vortices interact with the flow, there are three variants of their behavior: rotation, nutational oscillations, and unlimited elongation. This work describes the physical conditions of vortex transformation into filaments. The proportion of vortices that are stretching out into filaments in the World Ocean and some regions, redistributing the energy from mesoscale to submesoscale, is calculated.

Keywords: vortex, elongation, vortex line, filament

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It is known that in the ocean during the interaction between a mesoscale vortex and a flow, there is a regime when the vortex core is being stretched by this flow and the vortex actually ceases to exist as a localized formation. This variant of behavior corresponds to weak vortices in inhomogeneous flows. The calculation of energy showed that, when a vortex is elongating, its kinetic and available potential energies decrease [1].

Studies related to the transformation of an elliptical vortex during the interaction with the background flow originated in the works by Kirchhoff. Later, S.A. Chaplygin [2] and then S. Kida [3] showed three variants of behavior: rotation, nutational oscillations, and unlimited elongation. In the first two cases, the vortex remains a localized formation; in the last case, one of the axes increases unlimitedly and the other tends to zero. In the horizontal plane, such a vortex becomes similar to the vortex line (or filament).

The purpose of this work was to describe the physical conditions of elongation of 3D ellipsoidal vortices of the ocean into filaments and to estimate the proportion of mesoscale vortices being stretched into filaments, redistributing thereby the energy from mesoscale to submesoscale.

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In the works [1, 4–9], the theory of evolution of 3D-ellipsoidal barocline vortices under the action of flows was developed. For barotropic flows $\vec{u}_b = (u_b, v_b, 0)$ with a linear dependence of the flow velocity on the horizontal coordinates,

$$\vec{u}_b = (u_b, v_b, 0) = \begin{cases} u_b = u_0 + ex - \gamma y \\ v_b = v_0 + \gamma x - ey, \end{cases}$$

the problem reduces to the time evolution of two horizontal semiaxes of the ellipsoid $a(t)$ and $b(t)$. Here, $U = (u, v)$ is the flow velocity, u_0 and v_0 are the components of the flow velocity at the vortex center $x = 0$, $y = 0$; (x, y, z) is the Cartesian right-handed coordinate system: the x- and y-axes are horizontal, the z-axis is vertical; the coefficients γ and e describe the spatial variability of the background flow, $\gamma = \frac{1}{2}\mathrm{curl}_z \vec{u}_b$ is the angular velocity of rotation of liquid particles in the background flow, and e is the coefficient of background flow deformation. Any barotropic coordinate linear distribution of the velocities can reduce to the above dependence by rotating about a vertical axis. The vertical semiaxis c is constant in barotropic flows. The dimensionless parameter ε characterizes the degree of the vortex elongation in the horizontal plane and is determined by the ratio of its horizontal scales $\varepsilon = \frac{a}{b}$. Without loss of generality, $\varepsilon \geq 1$. Works [1, 5–12] introduce the dimensionless parameter of vertical flattening of the vortex core: $K = \frac{N}{f}\frac{c}{\sqrt{ab}}$, f is the Coriolis parameter, and $N =$ const is the Brunt-Väisäla frequency.

We note that the requirement for the background flow to be barotropic is a matter of convention. The

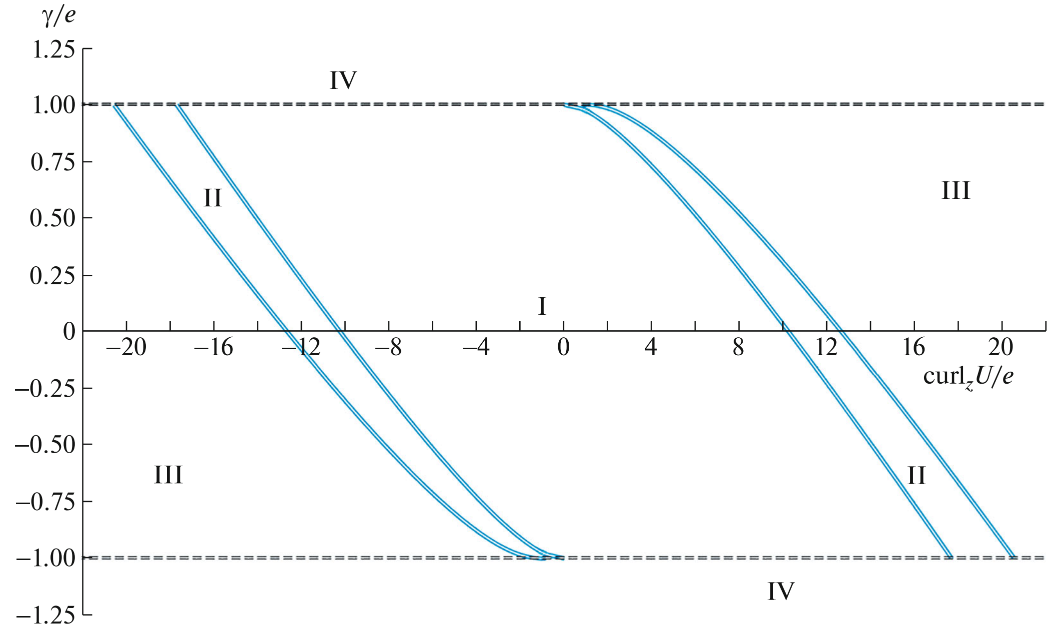

Fig. 1. Demonstration map of the zones of vortex behavior by the example of the value of $K = 0.4$ in the plane of parameters (γ/e and $\mathrm{curl}_z U/e$). Three regions are identified along the y-axis: for two regions $|\gamma/e| > 1$, there are only oscillating and rotational regimes (zone IV), the region extends to infinity; in the region $|\gamma/e| \le 1$, all three regimes, which are separated from each other by four curves starting pairwise at (0; ±1) and the point close to them, are allowed. As a result, the band $|\gamma/e| \le 1$ is divided into three symmetrical zones:
- internal (zone I), the regime of unlimited stretching of the vortex core is only obligatory;
- intermediate (zone II), the oscillating regime and the regime of unlimited stretching are allowed;
- external (zone III), all regimes: rotational, oscillating, and stretching are allowed.

vortex actively reacts to the flow on the horizons of the vortex core position and responds less actively or does not respond at all to the background flow above or below this layer. Therefore, to simplify, we considered flows that do not depend on the vertical coordinate at the levels of the vortex core position. It is not actually important what the flow above or below this layer is. Therefore, for mathematical simplicity, we extended the vertically homogeneous flow above and below the vortex core, making it the same as on the horizons of the core position. As a result, we obtained a model of barotropic flow through the whole ocean depth. In reality, we should take into account only the vertically averaged flow on the horizons of the vortex core position. This is what we will do in studying the evolution of vortices of the ocean near-surface layer, considering the average properties of the marine environment in the upper 200-m ocean near-surface layer.

The problem of the evolution of the vortex shape can be reduced to the system of two differential equations for the ratios of semiaxes and the angle of orientation θ formed by the large horizontal semiaxis a of the ellipsoid and the axis x. The solution of this system describes the evolution of a particular vortex that depends on the parameters e and γ of the background flow. The details of this conclusion are provided in [1, 5].

It can be shown that three dimensionless characteristics γ/e, σ/e, and K completely determine the evolution of the vortex when it is distorted by the flow under any initial conditions at ε and θ. Here, σ is the excessive potential vorticity of the vortex core compared to the potential vorticity of the background flow [1, 5]. It is convenient to replace σ/e with the parameter $\mathrm{curl}_z U/e$, since for circular vortices, σ and $\mathrm{curl}_z U$ are unanimously related to each other. The convenience of this set of numbers γ/e, $\mathrm{curl}_z U/e$, and K is as follows: γ/e is attributed exclusively to the characteristic of the background flow, $\mathrm{curl}_z U/e$ shows the relative intensity of the vortex, while K characterizes the flattening of the vortex core. The small values of $K < 1$ correspond to thin vortices, while the large values of $K > 1$ correspond to thick vortices. When the barotropic flow affects the vortex, the parameter K remains unchanged despite the distortion of the vortex core [1, 5]. The constancy of K for each vortex makes it possible to study the occurrence of each of the three behaviors of vortices on the plane of the parameters (γ/e and $\mathrm{curl}_z U/e$) (Fig. 1).

The boundaries of the zones are the bifurcation lines; when they intersect, a new behavior appears or the existing behavior of the vortex vanishes. We are interested mainly in zone I; here, the most important property is the limited vortex intensity. It is associated with relatively weak vortices that do not survive in inhomogeneous flows, stretching out into vortex fila-

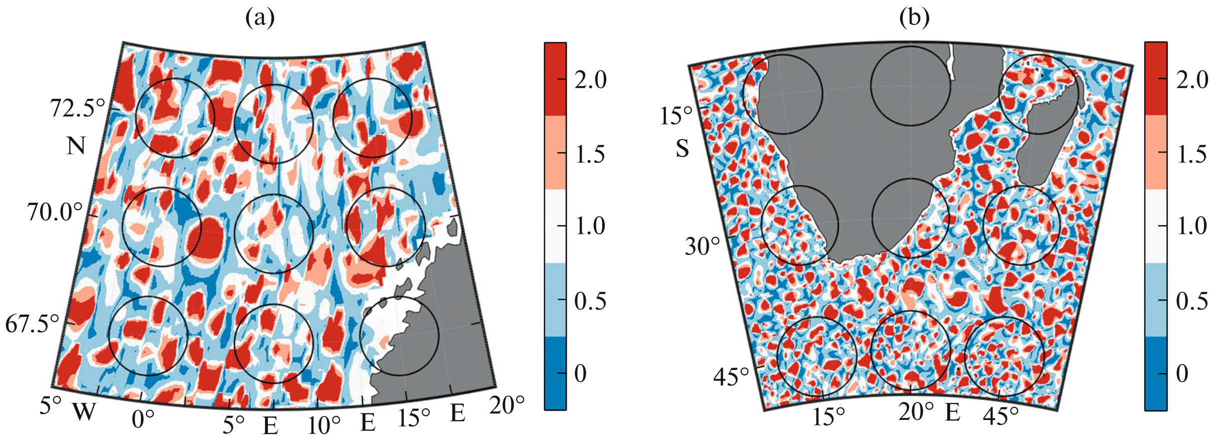

Fig. 2. Distributions of the values $|\gamma/e|$ (a) in the Lofoten basin and (b) in the region of the Agulhas flow. Red regions with $|\gamma/e| > 1$ correspond to the zones where unlimited extension of vortices is forbidden; in the blue regions with $|\gamma/e| < 1$, unlimited extension of vortices is allowed. The initial data have a spatial resolution of 0.25°, and smoothing was conducted by the moving-average method with window width of five cells. For comparison, we showed the circumferences with radii of (a) 100 km and (b) 500 km.

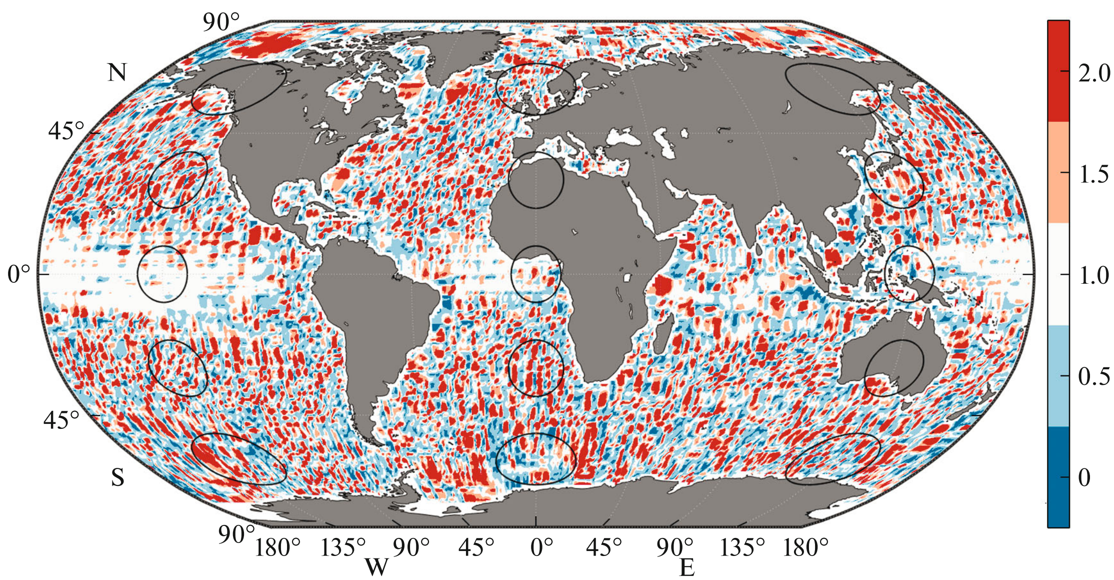

Fig. 3. Spatial distribution of the parameter $|\gamma/e|$ for the World Ocean. The initial data have a spatial resolution of 1°, smoothing was conducted by the moving-average method with the window width of ten cells. For comparison, we showed the circumferences with radius of 1000 km. The circumferences were drawn at latitudes 0°, 30°, and 60°, and longitudes of 0° and 135°.

ments. At an increase in the value of K, the boundaries of zones I and II "move" closer to the axis of ordinate γ/e, but never reach it. We may also show that at $K > 10$, the corresponding lines of the zone boundaries almost coincide, i.e., the internal part of zone I for the large values of K is almost limited by the black lines $K = 10$. Thus, the "heart" forms inside zone I, for which all vortices definitely stretch without limit regardless of the size vertically (parameter K) and regardless intensity of the vortex itself (parameter σ). The "heart" demonstrates the region of inevitable stretching of the Kirchhoff vortices from 2D hydrodynamics, which follows from our theory and corresponds to the generalization of Kida's work [3].

Now we consider these processes for the real ocean. We undertake calculations based on the data of the GLORYS12V1 global oceanic reanalysis [13].

Figure 2 presents the mapped calculations for the water areas of the Lofoten basin and the Agulhas current, and Fig. 3 shows data for the World Ocean as of June 10, 2010, with the averaged upper 200-m layer. In the vortex cores (red in color), stretching is forbidden, in the blue domains, vortex stretching is allowed. The proportion of the total area of the domains with permission for stretching for the Lofoten basin is 61%, while for the Agulhas basin, it is 50%. The total area of the domains in the World Ocean where vortices may stretch varies from 60 to 66%, depending on averaging of the data, which also exceeds the total area of the domains where vortex stretching is not allowed. We can show that the intra- and interannual variability of these properties is not pronounced just as seasonal variability, which means that the obtained estimates are stable.

The main conclusion of this work is during the evolution of mesoscale vortices against the background of the deforming flow, we should expect energy transfer from vortices to filaments, i.e., from mesoscale motions to submesoscale. This is a direct energy chain associated with unlimited stretching of vortices to filaments. According to the theoretical calculations, the vortex energy under a significant elongation of the core decreases by 20–60%. Since the physical system includes only a vortex and a flow, we can expect that the lost energy of vortices will redistribute back to the flow. If we return to representation of a vortex ensemble as geophysical turbulence where vortices are generated by the flow and then they interact with it at an energy level, then the phenomenon of energy return from turbulence to the flow can be termed a phenomenon of "negative viscosity" or an inverse energy cascade. Our work only touched upon the phenomenon of "negative viscosity." We showed its manifestation areas in the World Ocean (blue color in Figs. 2 and 3). Though the problem of vortex energy transfer throughout the size spectrum is not clear yet, the process of transformation of vortices into filaments should lead eventually to the integral redistribution of vortex energy from mesoscale to submesoscale (a direct energy cascade), while the decrease in energy during the process of vortex stretching returns energy to flows (inverse energy cascade or the negative viscosity phenomenon).

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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