High fidelity quantum logic gate for OAM single qudits

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Abstract

In this work, we propose a method for implementing multidimensional single-qudit gates for qudits based on light modes with orbital angular momentum (OAM) using the protocol for converting the OAM of light in the Raman quantum memory scheme. For different dimensions of qudits, we have shown that such transformations have an exceptionally high level of fidelity. We also compare quantum gates' properties of systems of different dimensions and find the optimal conditions for carrying out transformations in the protocol under consideration.

Today high-dimensional ($d-$dimensional) quantum systems (qudits) capture the attention of scientists. First of all, this is due to researchers are attracted by the possibility of increasing the information capacity of the channel – the amount of information that can be encoded in one physical carrier, that turns out to be very useful in the problems of quantum communication and quantum information processing [1]. Moreover, in quantum cryptographic protocols based on qudits, the security of the protocol appears to be higher, the larger the dimension of the system [2]. Nevertheless, there are still blind spots in the problem of efficient manipulation of multidimensional quantum states.

The orbital angular momentum (OAM) is one of the exciting resource for constructing a qudit since the OAM can take any integer values, which allows us to work in the Hilbert space of high dimension [3]. A significant factor is also that Laguerre– Gaussian modes with OAM show high stability and a relatively high decoherence time when propagating in a turbulent atmosphere [4].

To achieve the universality of quantum computation, it is necessary to be able to implement a universal set of quantum logic operations. Gates $\hat{X}_d^m$ and $\hat{Z}_d^m$ can be considered as single-qudit gates. The $\hat{X}_d^m$ gate performs modulo d addition of the value of the qudit with m, and the $\hat{Z}_d^m$ gate adds a relative phase to the terms of the superposition. Since the gate $\hat{Z}_d^m$ and its powers can be quite easily obtained using the Dove prism [5], we focused on constructing the gate $\hat{X}_d^m$. In this work, based on the protocol for converting the OAM of light in the Raman quantum memory scheme [6], we propose a method for implementing quantum single-qudit gates for various dimensions of qudits encoded in OAM numbers.

We have shown that the success probability of the conversion by adding an OAM quanta for the qudit dimension $d = 3$ increases with the value of the OAM l [7]. At the same time, in the case of subtracted OAM quanta ($m = -1, -2$), a higher probability can be achieved in the region of small values of l (see Fig. 1). The resulting conversion asymmetry is caused by the asymmetry of the mode overlapping functions and can be considered as an advantage of the protocol, which makes it possible to control in a wide range of values of l. Moreover, all transformations provide an exceptionally high level of fidelity (up to $F \geq 97$). Furthermore, we select the control parameter λ, which is the normalized relative shift of the beam waists of the driving and quantum fields. In particular, for each transformation a specific value of the parameter λ, provides high probabilities over a wide range of l values, could be calculated.

We have compared the characteristics of the $\hat{X}_d^m$ gates for different dimensions of qudits with the trivial case of a qubit. The values of probability and fidelity were taken into account, along with the potential gain from information capacity increased with the dimension of a qudit. We have evaluated the optimal qudit dimension for transformations. An estimation based on all these factors shows that working with qudits of dimensions $d = 3$ and $d = 4$ turns out to be preferable for performing quantum computations in the proposed protocol. Especially the drop in probability associated with an increase in the dimension of the system, is beaten by the high value of the information capacity of the channel.

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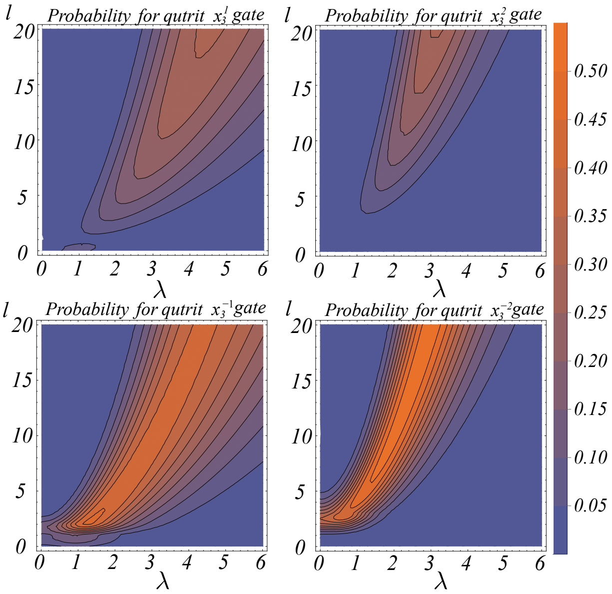

Figure 1: The probabilities of success of the qutrite gates $\hat{X}_3^1$, $\hat{X}_3^2$ (top), $\hat{X}_3^{-1}$, $\hat{X}_3^{-2}$ (bottom) depending on the values of l and of the parameter λ that controls the geometry of the modes.

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