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Penumbra–Umbra Area Ratio of Sunspots during Cycle-3 and Cycle Magnitude

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Abstract—Sunspot drawings of the Eimmart observatory are used to evaluate the fine structure of sunspots. The ratio of penumbra to umbra area of May–June 1703 and immediately after the Maunder minimum is found to be somewhat lower comparing to that of the Greenwich data. This result does not fit into the hypothesis that during the Maunder minimum the penumbra to umbra ratio was higher due to slower convection. We also compare distributions of sunspot group area of 1719–1720 and those of Greenwich database. Small sunspot groups are likely missing in the historical observations, so we suggest that the yearly magnitude of cycle-3 in 1719 was similar to that of cycles 13, 15, and 20.

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1. INTRODUCTION

Long-term sunspot indices are valuable to reconstruct magnetic flux, radial interplanetary magnetic field, spectral and total solar irradiance, and other parameters by the early eighteenth century (Dasi-Espuig et al., 2016; Lockwood et al., 2018; Wang and Lean, 2021). However, in the pre-Greenwich era (before 1874) number of days with sunspot records decreases (Vaquero and Vazquez, 2009; Svalgaard and Schatten, 2016; Vaquero et al., 2016), resulting in several versions of the sunspot time series. Uncertainties in historical observations force researchers to look for alternative ways to clarify the level of solar activity in the past. For instance, statistics of active-day fractions (Kovaltsov et al., 2004; Svalgaard and Schatten, 2017; Willamo et al., 2018), paleoreconstructions, auroral data, etc. (Rahmanifard et al, 2017; Larionova et al., 2020; Ogurtsov, 2019; Stangl and Ulrich, 2021). Another valuable parameter that makes it possible to indirectly clarify the activity of the Sun is the fine structure of sunspots. In various works (Waldmeier, 1939; Jensen et al., 1955; Antalova, 1991; Jun et al., 2006; Carrasco et al., 2018), researchers estimate the ratio of penumbra and umbra area (P/U and vice versa, umbra/penumbra), the ratio of the total and umbral area (W/U and vice versa, U/W), penumbra–umbra radius ratio, etc. These ratios are related to sunspot contrast which modulates variations of the total and spectral irradiance (Solanki, 2003) and, in turn, U/W anti-correlates with the Northern hemispheric temperature anomalies on Earth (Hoyt, 1979). Hypothesizing that an increase in convective velocities entails an increase in turbulent pressure on sunspots, Hoyt and Schatten (1997) suggested a decrease of P/U as a consequence. They expected that pressure forces the penumbra to decrease, while the isolated umbra remains unaffected. Higher P/U values are associated with both slower convective velocity and dimmer Sun. Parker (1976) proposed that during the Maunder minimum the convective mode of the Sun was less effective in the generation of the magnetic field. So, one has to expect higher P/U values, while after the minimum in the "normal" circulation mode this ratio decreases.

In this paper, we calculate the P/U ratio from the observational data stored at the Eimmart archive covering the Maunder minimum and the beginning of the normal circulation mode. We also estimated the distributions of the sunspot group area in historical and modern observations. The magnitude of cycle-3 is discussed.

2. DATA

National Library of Russia stores 58 volumes of the Eimmart observatory's archives. These manuscripts were recently analyzed by Hayakawa et al. (2021a; 2021b). Here, we consider three sets of sunspot areas from two drawings by Maria Eimmart (1703) of 30 and 31 May 1703, drawings by Eustachio Manfredi (1703) of 28 May–29 June 1703, and drawings by Johann Christoph Muller (1719–1720) of 20 April 1719–27 May 1720.

At the end of May 1703, Johann Heinrich Hoffman reported a sunspot group consisting of one large and a

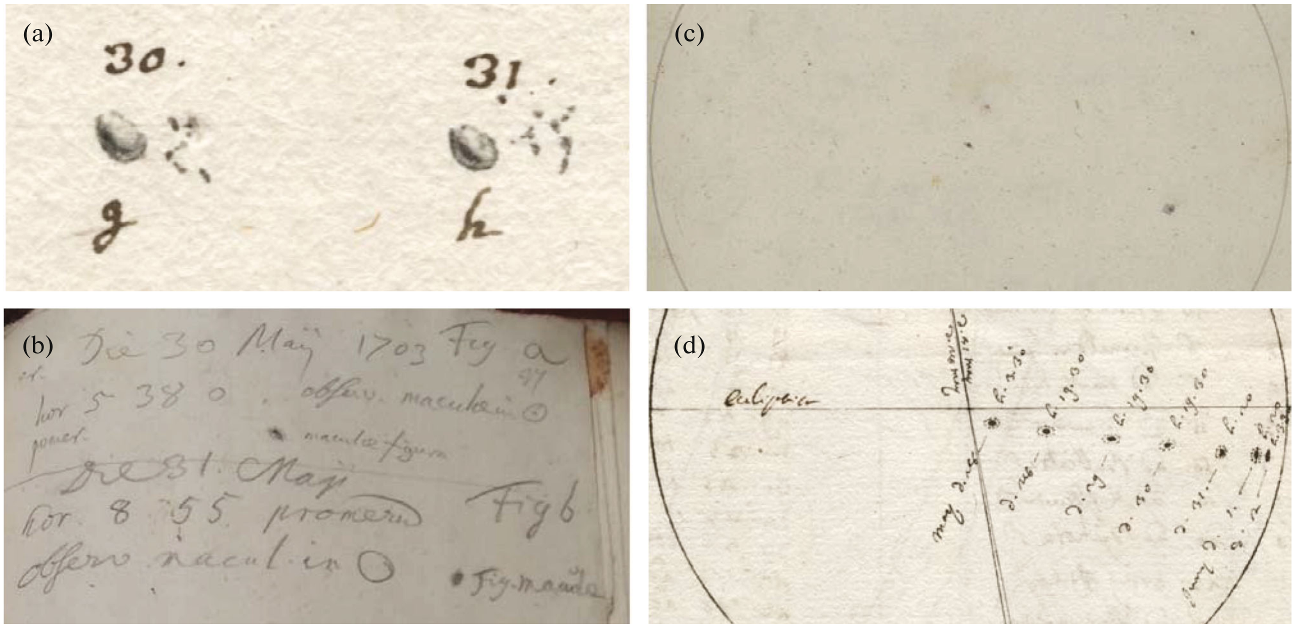

Fig. 1. Examples of sunspot drawings of May 1703 by Johann Heinrich Hoffman (a), Maria Eimmart (b and c), and Eustachio Manfredi (d), (fond 998 of National Library of Russian).

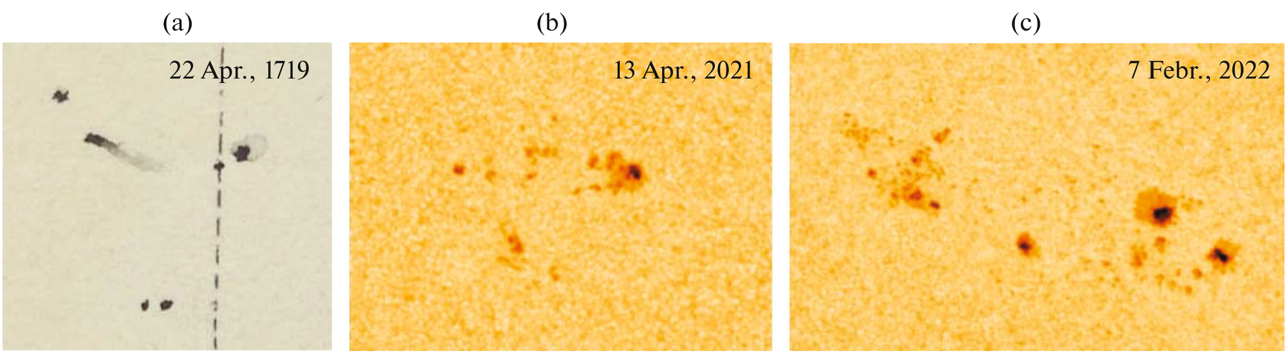

Fig. 2. (a) Example of sunspot drawings by Johann Muller (fond 998 of National Library of Russian), (b and c) HMI/SDO intensitygrams (Courtesy of NASA/SDO and the AIA, EVE, and HMI science teams).

myriad of small spots (Fig. 1a). Unfortunately, Hoffman did not distinguish between umbra and penumbra of sunspots. Maria Eimmart and Eustachio Manfredi, observed the same sunspot group and sketched it schematically showing only the largest sunspot of the group (Figs. 1b –1d).

The album of drawings by Johann Muller provides the fine structure of sunspots. However, some penumbra adjoins the umbra on one side, i.e., does not surround it. One of the examples is shown in Fig. 2a. Although, the penumbra of the two spots looks like spreading ink, these peculiar structures were drawn by Muller over several consecutive days, suggesting that both are not artifacts. Indeed, modern HMI intensitygrams occasionally exhibit similar forms of the penumbra. In Figure 2b, the penumbra follows the umbra, while in Fig. 2c the penumbra is located to the side of the umbra.

For each object, its area in pixels was calculated as the sum of pixels with intensity exceeding or falling below a certain threshold value. For each drawing, and sometimes individually for an object, the threshold was set individually due to the difference in the texture and color of the images. On the schematic drawings by Manfredi, the penumbra border is shown by a dotted curve. Here, the penumbra was filled in using a graphical editor. Then sunspot areas on the solar disk were estimated in microhemispheres (msh), the correction for foreshortening toward the limb was taken into account according to Meadows (2002), for details Vokhmyanin and Zolotova (2018).

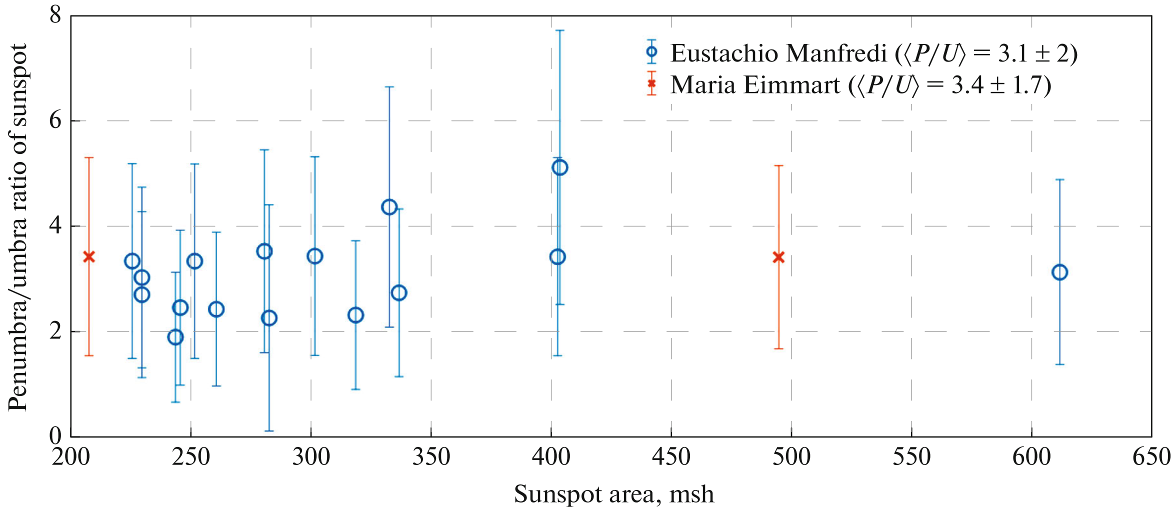

Fig. 3. Penumbra-to-umbra area ratio as a function of total sunspot area for Manfredi and Eimmart observations.

For a sunspot group, we calculate the P/U as the ratio of the total area of the penumbra of the group to the total area of the umbra. To evaluate the P/U ratio, we do not introduce a limit on the distance from the central meridian. The largest angular distance from the center of the disk to a sunspot is 66° for Manfredi and 69.8° for Muller. Only three sunspot groups by Muller for which the P/U ratio was calculated have an angular distance of more than 60°.

3. RESULTS

3.1. Ratio of Penumbra and Umbra Area

Figure 3 shows the P/U ratio defined from the drawings by Eimmart and Manfredi of 1703. Recall that here P/U are calculated for the individual sunspots. Since the original sunspot drawings are rather schematic, we assumed that the uncertainty of sunspot areas is ~30%. Despite the schematic style of the drawings (Fig. 1), the P/U ratios are similar for both observers. The average P/U of Manfredi's observations is 3.1 ± 2 (CI 95%: 1.9–5.1). Due to lack of the data points, we estimate the mean and standard deviations of P/U using the Monte-Carlo simulations. For each sunspot, we generate additional points from the normal distributions with mean value equal to P/U of that sunspot and sigma equal to the errors of that P/U value. Two observations by Eimmart (red in Fig. 3) provide P/U = 3.4 ± 1.7. Analyzing historical drawings, for the period of 1660–1709 Carrasco et al. (2018) found P/U to be 3.7, which is consistent with the ratios obtained above.

Also, W/U of sunspots is known to depend on the sunspot size (Antalova, 1971) and type (Tlatov, Riehokainen and Tlatova, 2019). For the largest sunspot of the group W/U ≈ 5.3, while for the remaining sunspots W/U ≈ 3.3. A similar trend is likely to exist in P/U. Note, that Manfredi drew the largest sunspot of the group, which is trailing sunspot (Fig. 1).

Figure 4 shows the P/U ratio defined from the observations by Muller after the end of the Maunder Minimum. Here, we calculate P/U for the whole set of sunspot groups (in blue color) and for a subset of sunspot groups (in red), which excludes objects with a peculiar structure, when the penumbra adjoins the umbra on only one side. The penumbra area of these peculiar sunspot groups and hence the P/U are smaller. For the whole set of sunspot groups (with regular and peculiar penumbra), the average P/U = 3.5 and median is 2.6 (CI 95%: 0.5–11.2, probability density function of Muller's observations has not a normal distribution). Sunspot groups with only regular penumbra structure, surrounding the umbra (Fig. 4), usually show higher values, the average of P/U = 4 and median is 3.4 (CI 95%: 1.9–8.5). Notice, that Galilei's observations at the beginning of the seventeenth century gave P/U = 3.8, while very schematic drawings by Cigoli of the same calendar dates gave 2.1 (schematic style of sunspot drawing tends to reduce P/U) (Vokhmyanin et al., 2021). So, we suggest that before, during, and after the Maunder minimum the P/U ratio has not changed dramatically.

Using sunspot photographic images taken with the 40 cm Vacuum Newton Telescope at the Teide Observatory, around the maximum of cycle 21 Brandt et al. (1990) estimated P/U = 4.17 for small spots and 3.13 for large spots. Later, analyzing the Greenwich Royal Observatory data of 1874 through 1976, Hathaway (2013) revealed that the P/U ratio varies from ~3 to 9, and on average this ratio increases from 5 to 6 as the total group area rises from 100 to 2000 msh. This ratio does not vary significantly with latitude or with the phase of a cycle. Later, these findings have been con-

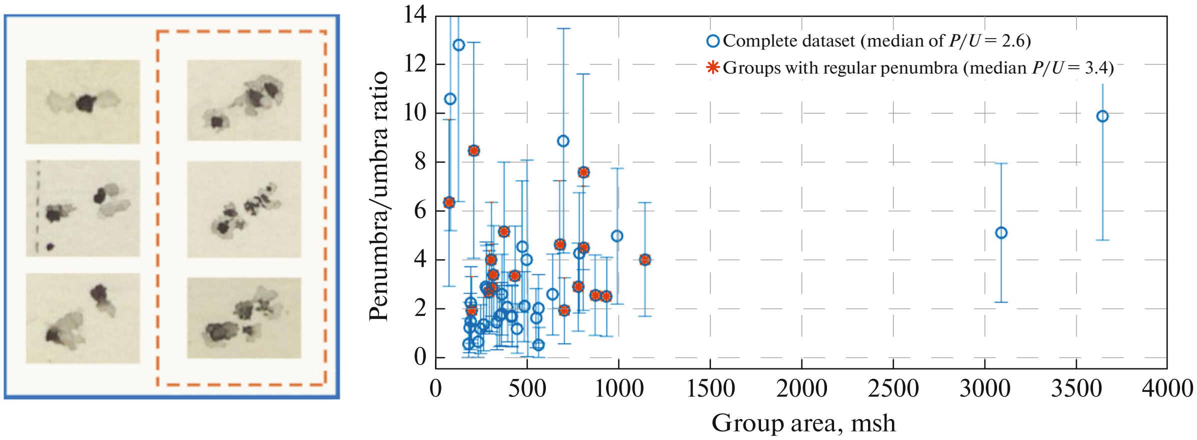

Fig. 4. Examples of sunspot groups drawn by Muller and their penumbra-to-umbra area ratio as a function of total sunspot group area.

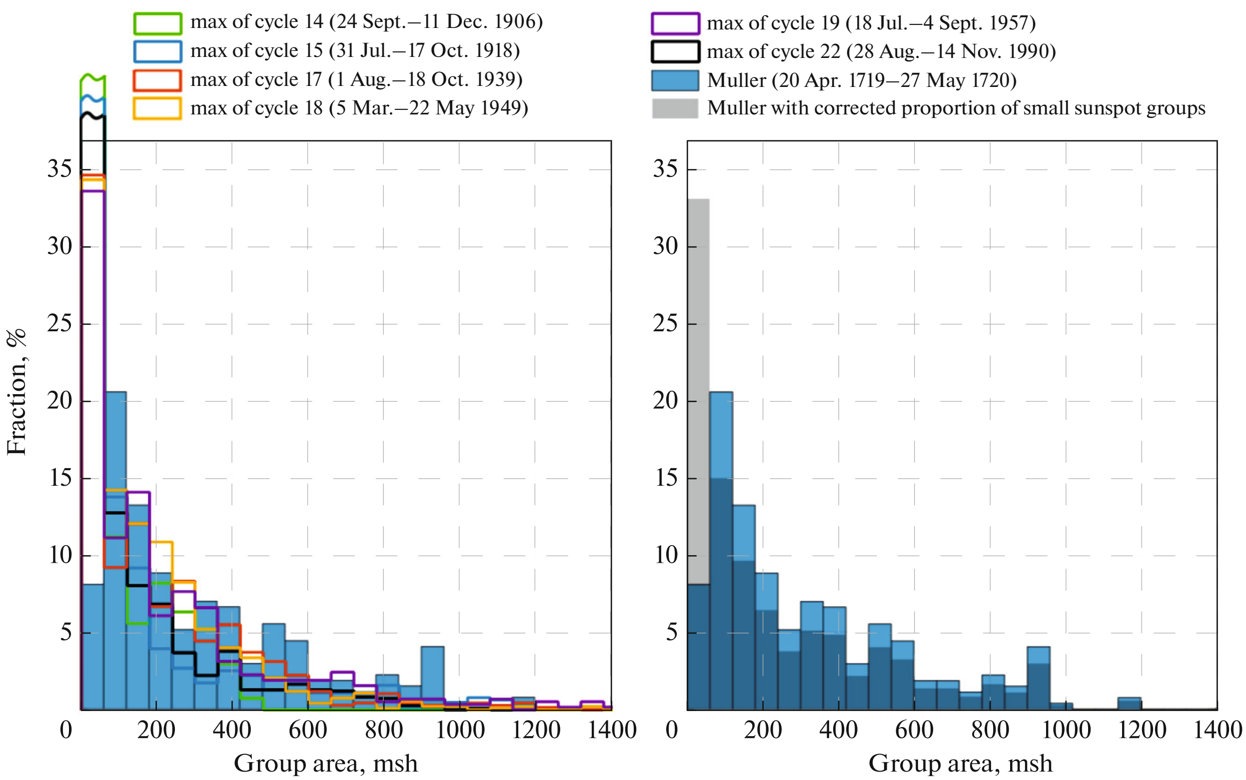

Fig. 5. Fractional distribution of sunspot group areas of Muller's observations and of RGO database.

firmed by Kodaikanal white-light data (Jha et al., 2019).

P/U ratios near the Maunder minimum were lower as compare to the modern epoch. This may be the result of seventeenth-century observations being inferior in accuracy to twentieth-century observations. Recall that if the convective velocities were lower during the Maunder minimum, then the P/U ration would have to be higher compared to the modern estimations.

3.2. Magnitude of Cycle-3

Another issue is related to the magnitude of cycle-3 of 1712 ± 2 through 1723 ± 2. Observations by Muller cover a period near the maximum of this cycle. Figure 5a shows the fractional distribution of sunspot group areas defined from Muller's drawings and modern observations. For comparison of the modern and historical reports, we use RGO/USAF/NOAA database from 1900 to 2016 and select periods near the solar maxima. Among all data, cycles 17, 18, and 19 have the smallest portion of sunspot groups with an area <60 msh, about 35%, while cycles 14, 15, and 22 have the largest portion of small groups: cycles 15 and 22–55% and cycle 14–65%. Here, we hypothesize that the historical abundance of sunspot groups at the cycle maxima should be similar to the modern era with a fraction of small groups varying from ~35% to 65%. If the distribution of sunspot group areas is the same, then Muller's reports might miss from ~25 to 60% of small sunspot groups. Figure 5b shows the original and corrected (small spots number is ~4 times higher resulting in overall increase by up to 25%) distributions. According to Group Number series version 2.0, the yearly mean number of groups of 1719 is 6.16. Following the hypothesis that Muller's observation may lack at least 25% of small sunspots, the corrected yearly number of groups should be 7.7. This magnitude of cycle-3 in 1719 is comparable to the maxima of cycles 13, 15, and 20.

4. CONCLUSIONS

In this work, we analyze sunspot observations stored in the Eimmart archives. Estimation of the fine structure of sunspots of May 1703 provides that the penumbra to umbra ratio is 3.1 ± 2 (drawings by Manfredi) and 3.4 ± 1.7 (drawings by Eimmart). These values agree with those obtained by Carrasco et al. (2018) between 1660 and 1709. Observations by Muller of 1719–1720 can be divided into two types of sunspot groups: peculiar penumbra adjoining the umbra at only one side and regular penumbra surrounding the umbra. The whole set of sunspot groups yields the penumbra to umbra ratio equal 3.5 (mean) and 2.6 (median), while groups with only regular penumbra gave 4 (mean) and median is 3.4. We conclude that these quantities are comparable during and immediately after the Maunder minimum, but are lower compared to RGO data (1900–2016), whose average ratio is ~5.5 (Hathaway, 2013). These findings do not fit into the hypothesis of slower convective velocity during the Maunder minimum.

If the ratio of large and small spots is a constant quantity through the centuries, Muller's drawings may lack from ~25 to 60% of small sunspot groups. In this case, the yearly magnitude of sunspot groups of cycle-3 in 1719 must be at least 7.7 which is comparable to the maxima of cycles 13, 15, and 20. This result is consistent with the conclusion by Ogurtsov (2013) that the uncertainty of the sunspot numbers in the 17–18th centuries is close to 30%, and in some years, it may reach factor 2 or more.

We use HMI/SDO intensitygrams (https://sdo.gsfc.nasa.gov), data of the Royal Greenwich Observatory (RGO/USAF/NOAA: https://solarscience.msfc.nasa.gov/greenwch.shtml), drawings from the Eimmart archives of fond 998 of National Library of Russian (http://nlr.ru/eng), and the new Group Number series version 2.0 from WDC–SILSO (http://www.sidc.be/silso/groupnumber).

CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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