Volume 6 Number 1

1996
ISSN 1060-9881

NOVA JOURNAL OF MATHEMATICS, GAME THEORY, AND ALGEBRA

Nova Science Publishers, Inc.

Nova Journal of Mathematics, Game Theory, and Algebra
Volume 6, Number 1, pp. 7-14, 1996

ISSN 1060-9881

SOME GAME-THEORETICAL MODELS OF CONFLICT IN FINANCE

VLADIMIR M. BURE AND OLEG A. MALAFEYEV
Faculty of Applied Mathematics
St. Petersburg State University
Russia

1. Introduction

Two game-theoretical models of conflict are considered in the paper. The first model gives approach to the solving of a pricing problem. The problem of pricing concerns of the choosing of a price function in order to maximize a certain criterion. The problem has been studied in [1],[6].

In this paper we give another approach based on making use of distribution of a random vector. We can interpret this probability measure as a distribution of the buyers or as distribution characterizing imperfectness of information about the buyers.

The second model is concerned of a problem of repeated partnerships with imperfect information. This problem has been studied in [2],[7]. In our paper some results of [2] are extended to the case of $n-$person games.

2. Pricing problem

Let some agent (seller, player 0) emits stock of some kind. Let a price $p(u)$ of one share of the stock emitted by the player 0 is a function of quantity shares u bought by a buyer (player) $i, i = \overline{1:n}$.

Let $u, u \in N$, here $N = \{0, 1, 2, 3, ...\}$ is the set of nonnegative integer numbers. Let $p(u)$, $p(u) \in N$, be a monotone nonincreasing function for which there exist constants p_0, p_1 such that for any $u \in N$ the following inequalities are valid:

$$p_0 \leq p(u) \leq p_1. \tag{1}$$

Let F be the set of all such functions.

Assume that there are n, $i = \overline{1:n}$, buyers. The buyer i, $i = \overline{1:n}$, chooses quantity shares u_i, $i = \overline{1:n}$, that he will acquire under the condition that seller's price function is $p(u_i) \in F$.

Every buyer has his own utility function $S_i(u_i)$, $i = \overline{1:n}$. Let an utility function $S_i(u_i)$ be a nonnegative concave monotone nonincreasing function and $S_i(0) = 0$, $\lim S_i'(u_i) = 0$ for $u_i \to \infty$, $i = \overline{1:n}$.

It is easy to see that the payoff function $H_i(u_i)$ of the buyer (player) i is determined by the following formula:

$$H_i(p, u_i) = S_i(u_i) - u_i p(u_i), \quad i = \overline{1:n}. \tag{2}$$

The payoff function H_0 of the seller (player 0) is determined by the following formula:

$$H_0(p, u_1, ..., u_n) = \sum_{i=1}^{n} u_i p(u_i). \tag{3}$$

Thus we have $(n+1)-$person game Γ. Payoff functions of players are defined by formulae (2),(3). It is easy to see that the strategy set of the player (buyer) $i, i = \overline{1:n}$, is the set

$$X_i = \{u_i \in N : S_i(u_i) - u_i p_0 \geq 0\}.$$

It is evident that the set X_i is finite.

The strategy set of the player 0 (seller) is the set

$$F_0 = \{p(u) : p(u) \in F, u \in \bigcup_{i=1}^{n} X_i\}.$$

The set F_0 is also finite.

Thus we have

$$\Gamma = (I; F_0, X_i, i = \overline{1:n}; H_0, H_i, i = \overline{1:n}),$$

here $I = \{0, 1, ..., n\}$.

The player 0 (seller) is called a leader in this game and other players are forced to find their best reactions. We assume that the buyer i has no information about other buyers and he knows only the price function taken by the seller.

We extend the Stackelberg equilibrium concept for $(n+1)-$person game.

We define the point $(p^*, u_1^*, ..., u_n^*)$ to be the point of equilibrium in game Γ if the following conditions are hold:

$$BR = \{(q, x_1, ..., x_n) : q \in F_0, x_i = \arg \sup_{u_i \in X_i} H_i(q, u_i), i = \overline{1:n}\},$$

$$H_0(p^*, u_1^*, ..., u_n^*) = \sup_{(q, x_1, ..., x_n) \in BR} H_0(q, x_1, ..., x_n).$$

Theorem 1. *A point of equilibrium exists in the game* Γ.

This theorem follows from the finitness of the players strategy sets.

Remark. Let us suppose $p(u)$ to be a continuous price function defined on the interval $[0, \infty)$. Then the strategy sets being compact guarantees the existence of maximization problems decisions. This implies the existence of an equilibrium in the game Γ as well.

Now we consider a concrete realization of the above presented price analysis. The principle difficulty in the problem of applicating our price analysis is the imperfectness of the information that the seller posesses about buyers' utility functions. Our purpose now is the solving of this problem.

Let

$$\Phi(u) = up(u) \tag{4}$$

be a loss function for a buyer.

In the case $u_1 < u_2$ it is appropriate to suppose the loss function to be satisfying the inequality

$$\Phi(u_1 + h) - \Phi(u_1) \geq \Phi(u_2 + h) - \Phi(u_2)$$

for all $h > 0$.

Therefore we can assume that a loss function $\Phi(u)$ is concave. We also assume that each buyer i can be characterized by two parameters: let C_i be a player i's conjectural profit from one share and let A_i be a desirable level of a total income. Thus the parameter $u_i' = A_i/C_i$ is a desirable quantity of shares for the buyer i.

We can say that the parameter C_i characterizes a degree of optimism of the buyer i and the parameter A_i characterizes a degree of intention (and financial capabilities) of buyer i.

Therefore one can assume that the utility function is defined as follows: $S_i(u)$ is equal to $C_i u$ if $u \leq u_i'$ and $S_i(u)$ is equal to A_i if $u > u_i$.

It is evident that

$$u_i' = \arg \sup_{u_i \in X_i} H_i(\Phi, u_i),$$

here $H_i(\Phi, u_i) = S_i(u_i) - \Phi(u_i)$.

Now we make the following assumption: every buyer acquires u_i' shares if and only if

$$H_i(\Phi, u_i') \geq qS_i(u_i'),$$

here q is a rate of interest, $0 < q < 1$.

Let $Q_{u', A}(x, y)$ be a probability measure which is a distribution of random vector (u', A). We can interpret this probability measure as a distribution of buyers or as a distribution characterizing imperfectness of information about buyers and in this case there can be only one buyer in the market.

The expected payment for the seller from one buyer is

$$H^0(\Phi, Q_{u'}, A) = \int_{\{(x,y):y-\Phi(x)\geq qy\}} \Phi(x) dQ_{u'}, A(x, y), \qquad (5)$$

here $\Phi(x) = xp(x)$, $p(x)$ is a price function.

Thus we have the maximization problem for obtaining the optimal price function

$$\Phi^* = \arg\sup H^0(\Phi, Q_{u'}, A),$$
$$p^*(u) = \Phi^*(u)/u, \qquad (6)$$

the function Φ^* is supposed to be nondecreasing concave and $p^* \in F$.

If the support of the measure $Q_{u'}, A$ is finite we have a finite set of admissible loss functions $\Phi(u)$. In this case the problem (6) can be solved by a finite item-by-item examination.

3. Repeated partnerships

Let players emit stock of different kinds during consecutive time intervals. Let player $i, i = \overline{1:n}$, has m_i different decisions about emitting stock inside each interval. The decisions made by all players befor time following interval define payoffs of each player for this interval.

Let matrix $A(k) = (a(k; i_1, \dots, i_n)), i_1 = \overline{1:m_1}, \dots, i_n = \overline{1:m_n}$, define payoffs of a player $k, k = \overline{1:n}$, in a game $A = (A(1), \dots, A(n))$.

Let $I = \{(i_1, \dots, i_n)\} : i_1 = \overline{1:m_1}, \dots, i_n = \overline{1:m_n}\}$ be a set of all strategic n-tuples of the game $A = (A(1), \dots, A(n))$.

We define correlated strategis as follows: the players choose a n-tuple $(i_1, \dots, i_n)$ with probability $p(i_1, \dots, i_n)$. The outcome of the game is defined by distribution P :

$$P = (p(i_i, \dots, i_n)), (i_1, \dots, i_n) \in I, p(i_1, \dots, i_n) \geq 0,$$

$$\sum_{(i_1, \dots, i_n) \in I} p(i_1, \dots, i_n) = 1. \qquad (7)$$

An income of a player $k, k = \overline{1:n}$ is an expectation :

$$h(A(k), P) = \sum_{(i_1, \dots, i_n) \in I} a(k; i_1, \dots, i_n) p(i_1, \dots, i_n).$$

As the distribution P we choose the Nash arbitration solution, the point of compromise [3,4,5] etc. We suppose that players use public randomizing device with distribution P for finding strategic $n-$tuple. But players can have incentives to deviate.