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Journal "Regular and Chaotic Dynamics"

The International Scientific Workshop

"Recent Advances in Hamiltonian and Nonholonomic Dynamics"

Book of Abstracts

15–18 June 2017
Dolgoprudny, Russia

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Contents

The phase space of a mechanical system with singularities

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In the present report, we discuss some approaches to build the geometrical image of phase space over the manifold with singularities. The start point of our consideration is the real mechanical model of double pendulum with restriction.The rod lengths ratio is 2:1.

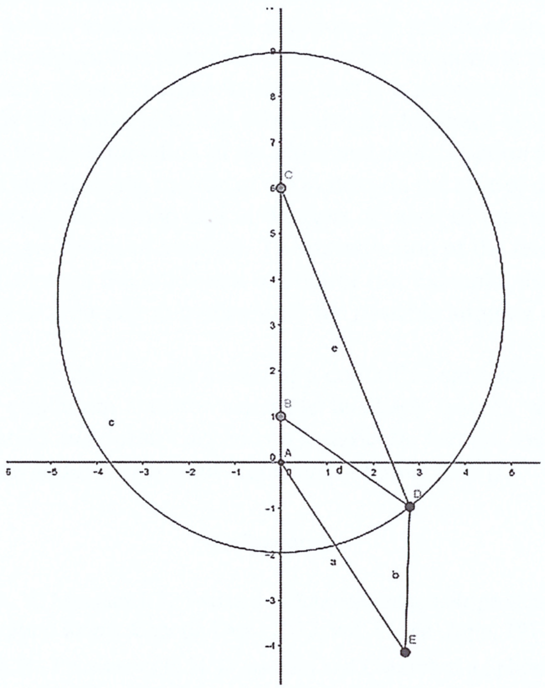

Fig. 1. Double pendulum with restriction

A thread of fixed length holds the end of a double pendulum, with their own ends fixed on the vertical axis. It gives that the trajectory of the pendulum end is an ellipse. Let us start with the length of the thread long enough for motion through the vertical. The critical length is exactly when the rods of the pendulum merge in the vertical position. Right before the critical value

the motion of the pendulum has the following character: near the vertical, the second rod leaves the elliptical trajectory and leaps to a symmetrical position on restriction with the hit. When the length of the thread goes shorter, the point of leaving goes closer to the vertical position. Finally, in the critical case the points of leaving and hitting coincide and we observe the smooth symmetrical motion near the vertical. All this is experimental data.

Now let us consider the configuration space of the pendulum. In the neighborhood of the equilibrium state the space is two one-dimensional manifoldsin R^4 with one point of intersection. It is because there are two possible positions of the second rod on the thread: before and after the first one. The intersection in the general case is transversal and so the real trajectory of the system on the configuration space has the angle without stop. It means that on any geometrical image of phase space, as some sort of a bundle, the projection of the evolution trajectory is not a smooth trajectory on the base.

Moreover, it easy to calculate the length of the thread and the positions of their fixed ends such that the configuration space is the two lines with a contact point of nonzero order. Now, any real trajectory is smooth, but we have geometrical uncertainty, which contradicts dynamical certainty. Therefore, any geometrical model of the phase space should guarantee the last.

In our investigation, we have considered several approaches to this problem. Namely, the Sikorsky space [1] and Diffiety space [2]. Both approaches give different geometrical descriptions of the phase space as a bundle over the singular manifold and some forms of a Hamiltonian vector field on them. However, both approaches in the cases of transversality require that the solution of the differential equation in generalized sense must have zero velocity at a singular point. In the case of geometrical uncertainty, they do not give the resolution of it.

To explain the observed effect, we should suppose that the bundle leaf over a singular point have more degrees of freedom than one. Maybe, for example, the generalized solution of the Hamiltonian vector field poses a parameter which is not time but some reparametrization of it.

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